Wavelet-Based 3D Compression Scheme for Very Large Volume Data

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Background

- Volume data are often very large.
  - Several hundred MB ~ several dozen GB
- Huge volume data place considerable demands on run-time memory space.
- Such data sets need special treatment for effective manipulation.

 آلاف

Can we interactively visualize a 2GB volume data set on my computer with 128MB of main memory?
Volume Rendered Visible Man
Visible Human Dataset from NLM

- CT, MRI and RGB cryosection images
- Visible Man
  - Axial scans: 1mm interval, over 1870 cross-sections
  - Cross-sectional images: 512 x 512
  - Ex) Frozen CT 512 x 512 x 1871 x 2 bytes = 935.5MB
- Visible Woman
  - Axial scans: 1/3mm interval

总收入数据集达到15 GB到40 GB。
Research Goal

Design a compression scheme for very large volume data suitable for developing interactive applications!

- Load the whole very large volume data, say 2GB, into a main memory of moderate size, say 128MB
- Allows users feel as if they have a main memory of larger sizes
- Helps develop interactive applications for very large volume data on PCs or workstations with limited memory
Two Important Design Factors

- High compression ratio
  - Achieve the best compression rate with minimal distortion in the reconstructed images?
  - Often impose some constraints on random access ability

- Fast run-time random access ability
  - The access patterns in interactive applications change in somewhat complicated ways.

☞ Compromise between these two factors!
Related Work

- Ning et al. [Vis. 93]
  - Vector quantization
- Muraki [Vis. 92, CG&A 93]
  - 3D Wavelet compression
- Ghavamnia et al. [Vis. 95]
  - Laplacian pyramid
- Yeo et al. [IEEE TVCG 95]
  - DCT-based 3D compression
- Thoma et al. [IEEE Mult. 97]
  - Experimental results on 2D compression of VH
Wavelets

- A mathematical tool for representing functions hierarchically

  ✓ Allow functions to be represented in multi-resolution forms
  ✓ Offer a mathematical basis for data compression
  ✓ Have recently had a great impact in computer graphics
1D Haar Wavelet Transforms

\[ X^3 = [107, 109, 110, 110, 108, 107, 106, 105] \]

\[ X^2 = [108, 110, 107.5, 105.5] \]

\[ Y^2 = [-1, 0, 0.5, 0.5] \]

\[ X^1 = [109, 106.5] \]

\[ Y^1 = [-1, 1] \]

\[ X^0 = 107.75 \]

\[ Y^0 = 1.25 \]

Decomposition

\[ c^H = (c^J - c^S) \setminus \frac{\mathbb{S}}{\mathbb{S}} \]

\[ c^\Gamma = (c^J + c^S) \setminus \frac{\mathbb{S}}{\mathbb{S}} \]

Reconstruction

\[ c^S = c^\Gamma - c^H \]

\[ c^J = c^\Gamma + c^H \]
2D Haar Wavelet Transforms

Decomposition

\[ c_{LL} = \frac{c_1 + c_2 + c_3 + c_4}{4} \]
\[ c_{HL} = \frac{c_1 - c_2 + c_3 - c_4}{4} \]
\[ c_{LH} = \frac{c_1 + c_2 - c_3 - c_4}{4} \]
\[ c_{HH} = \frac{c_1 - c_2 - c_3 + c_4}{4} \]

Reconstruction

\[ c_1 = c_{LL} + c_{HL} + c_{LH} + c_{HH} \]
\[ c_2 = c_{LL} - c_{HL} + c_{LH} - c_{HH} \]
\[ c_3 = c_{LL} + c_{HL} - c_{LH} - c_{HH} \]
\[ c_4 = c_{LL} - c_{HL} - c_{LH} + c_{HH} \]
Wavelet-Based Compression

• Idea
  - Decompose an input data set through wavelet transforms.
  - Sort the wavelet coefficients in order of decreasing magnitude.
  - Given an error measure, delete as many coefficients with smaller magnitude as possible.

\[ f(x) = \sum_{i=1}^{m} c_i \cdot u_i(x) \Rightarrow \overline{f}(x) = \sum_{i=1}^{\overline{m}} \overline{c}_i \cdot \overline{u}_i(x) \quad (\overline{m} \leq m) \]

This is the best choice for orthonormal bases under the L2 norm.
Ex: 2D Image Compression

Decomposition

Reconstruction

5%

5%
3D Haar Wavelet Transforms

Decomposition

\[ C_{LLL} = \left( C_1 + 2C_2 + 3C_3 + 4C_4 + 5C_5 + 6C_6 + 7C_7 + 8C_8 \right) / 8 \]

\[ C_{LLH} = \left( C_1 + 2C_2 + 3C_3 + 4C_4 - 5C_5 - 6C_6 - 7C_7 - 8C_8 \right) / 8 \]

\[ C_{LHL} = \left( C_1 + 2C_2 - 3C_3 - 4C_4 + 5C_5 + 6C_6 - 7C_7 - 8C_8 \right) / 8 \]

\[ C_{LHH} = \left( C_1 + 2C_2 - 3C_3 - 4C_4 - 5C_5 - 6C_6 + 7C_7 + 8C_8 \right) / 8 \]

\[ C_{HLL} = \left( C_1 - 2C_2 + 3C_3 - 4C_4 + 5C_5 - 6C_6 + 7C_7 - 8C_8 \right) / 8 \]

\[ C_{HLH} = \left( C_1 - 2C_2 + 3C_3 - 4C_4 - 5C_5 + 6C_6 - 7C_7 + 8C_8 \right) / 8 \]

\[ C_{HHL} = \left( C_1 - 2C_2 - 3C_3 + 4C_4 + 5C_5 - 6C_6 - 7C_7 + 8C_8 \right) / 8 \]

\[ C_{HHH} = \left( C_1 - 2C_2 - 3C_3 + 4C_4 - 5C_5 + 6C_6 + 7C_7 - 8C_8 \right) / 8 \]
3D Wavelet Compression

- How do we encode and store the necessary information in a smaller number of bits?
  - Zerotree [Shapiro IEEE TSP 93, Said et al. IEEE Symp. Cir. & Sys. 93, …]
3D Haar Wavelet Decomposition

- Apply the 3D wavelet transform twice.
New Compression Scheme

Preprocessing
- Collection of 16x16x16 unit blocks

Run-time computation
- 3D wavelet decomposition
- (adaptive) ratio of non-zero coef’s
- Replacement of smaller coef’s by zeros
- Encoder

Wavelet-based encoding structure
Encoding Scheme

- Partition a 16x16x16 unit block into 64 cells of size 4x4x4
New Compression Scheme

Preprocessing
- Collection of 16x16x16 unit blocks

Run-time computation
- 3D wavelet decomposition
- Replacement of smaller coef’s by zeros
- (adaptive) ratio of non-zero coef’s
- Encoder
- 3D wavelet reconstruction
- Decoder
- Wavelet-based encoding structure
- Index (i,j,k)
- Voxel value
Reconstruction

\[
c_1 = c_{LLL} + c_{LLH} + c_{LHL} + c_{LHH} + c_{HLL} + c_{HLH} + c_{HHL} + c_{HHH}
\]
\[
c_2 = c_{LLL} + c_{LLH} + c_{LHL} + c_{LHH} - c_{HLL} - c_{HLH} - c_{HHL} - c_{HHH}
\]
\[
c_3 = c_{LLL} + c_{LLH} - c_{LHL} - c_{LHH} + c_{HLL} + c_{HLH} - c_{HHL} - c_{HHH}
\]
\[
c_4 = c_{LLL} + c_{LLH} - c_{LHL} - c_{LHH} - c_{HLL} - c_{HLH} + c_{HHL} + c_{HHH}
\]
\[
c_5 = c_{LLL} - c_{LLH} + c_{LHL} - c_{LHH} + c_{HLL} - c_{HLH} + c_{HHL} - c_{HHH}
\]
\[
c_6 = c_{LLL} - c_{LLH} + c_{LHL} - c_{LHH} - c_{HLL} + c_{HLH} - c_{HHL} + c_{HHH}
\]
\[
c_7 = c_{LLL} - c_{LLH} - c_{LHL} + c_{LHH} + c_{HLL} - c_{HLH} - c_{HHL} + c_{HHH}
\]
\[
c_8 = c_{LLL} - c_{LLH} - c_{LHL} + c_{LHH} - c_{HLL} + c_{HLH} + c_{HHL} - c_{HHH}
\]
Reconstruction Scheme

- All the necessary wavelet coefficients are first decoded from encoded unit block.
- The reconstruction formula is applied twice.

When a specific voxel is reconstructed,
- 15 decoding operations
- 14 +/- operations for reconstruction

When 64 voxels in a cell are reconstructed simultaneously,
- 64 decoding operations: one per voxel
- 24 +/- operations for reconstruction: 3 and 3/4 per voxel
Decoding Algorithm

- **Input**: a 16x16x16 encoded unit block and an index \((i, j, k)\)
- **Output**: the decoded value of the voxel with index \((i, j, k)\)

1. Find the cell \(C\) that contains the index \((i, j, k)\).
2. If the cell tag for \(C\) is 0, return 0. \([\text{case } 1]\)
3. Compute the relative index \((i', j', k')\) in \(C\).
4. If the bit-flag for \((i', j', k')\) is 0, return 0. \([\text{case } 2]\)
5. Count the number of preceding non-zero coefficients by table access.
6. Add the displacement to the offset to compute the correct address.
7. Access the appropriate data stream, and return the value. \([\text{case } 3]\)

Most cases of decoding belong in either [case1] or [case2] \(\Rightarrow\) Decoding is cheap!
Analysis of Compression Ratio

\[
\frac{1}{\rho} = \frac{2 + 64 + (8 + 2) \cdot 64 \cdot \alpha + 16^3 \cdot \gamma \cdot (\beta + 2 \cdot (1 - \beta))}{16^3 \cdot 2} \\
\rho \approx 0.008057 + 0.078125 \cdot \alpha + \gamma \cdot (1 - 0.5 \cdot \beta)
\]

\(\rho = \) compression ratio \\
\(\alpha = \) # of non-null cells / 64 \\
\(\beta = \) # of coef’s in one-byte stream \\
\(\beta = \) # of all non-zero coef’s used \\
\(\gamma = \) ratio of non-zero wavelet coef’s
Experiments

- **H/W**
  - SGI Octane with 175 MHz R10000 CPU

- **Test data**
  - Preprocessed fresh CT data of Visible Man
  - $512 \times 512 \times 1440 \times 2$ bytes $= 720$ MB
  - $92160 \ (= 32 \times 32 \times 90) \ 16 \times 16 \times 16$ unit blocks
Adaptive Distribution of Nonzero Coef’s

- $\bar{\gamma}$: a desired ratio of nonzero coef’s to be used
- Apply the wavelet transforms to each unit block $i$, and compute the ratio $r_i$ of nonzero coefficients.
- For the unit block $i$,
  
  allocate $n_i = \frac{r_i}{\sum_j r_j} \cdot 512^2 \cdot 1440 \cdot \bar{\gamma}$ coefficients.
- Compute the corresponding threshold value for each unit block.
## Compression Ratio

<table>
<thead>
<tr>
<th></th>
<th>3%</th>
<th>5%</th>
<th>7%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compressed Data Size (MB)</strong></td>
<td>26.31</td>
<td>38.27</td>
<td>48.98</td>
<td>63.69</td>
<td>85.99</td>
</tr>
<tr>
<td><strong>Compression Ratio</strong></td>
<td>27.4 : 1</td>
<td>18.8 : 1</td>
<td>14.7 : 1</td>
<td>11.3 : 1</td>
<td>8.4 : 1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.1156</td>
<td>0.1901</td>
<td>0.2481</td>
<td>0.3165</td>
<td>0.4014</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.6559</td>
<td>0.7327</td>
<td>0.7771</td>
<td>0.8185</td>
<td>0.8600</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0293</td>
<td>0.0481</td>
<td>0.0667</td>
<td>0.0946</td>
<td>0.1407</td>
</tr>
</tbody>
</table>

$\alpha = \# \text{ of non-null cells} / 64$

$\beta = \# \text{ of coef’s in one-byte stream} / \# \text{ of all non-zero coef’s used}$

$\gamma = \text{ ratio of non-zero wavelet coef’s}$
## Compression Error

<table>
<thead>
<tr>
<th>Errors in Voxels Values</th>
<th>SNR (dB)</th>
<th>3%</th>
<th>5%</th>
<th>7%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>22.6</td>
<td>25.9</td>
<td>28.6</td>
<td>32.0</td>
<td>36.4</td>
</tr>
<tr>
<td>PSNR (dB)</td>
<td></td>
<td>44.5</td>
<td>47.8</td>
<td>50.4</td>
<td>53.8</td>
<td>58.2</td>
</tr>
<tr>
<td>Errors in Normals</td>
<td>Skin (deg)</td>
<td>15.4</td>
<td>11.4</td>
<td>8.6</td>
<td>6.0</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>24.7</td>
<td>19.0</td>
<td>15.0</td>
<td>10.8</td>
<td>6.7</td>
</tr>
</tbody>
</table>
## Random Access Time

<table>
<thead>
<tr>
<th></th>
<th>Uncompressed</th>
<th>3%</th>
<th>5%</th>
<th>7%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Random</td>
<td>2.18</td>
<td>7.75</td>
<td>8.20</td>
<td>8.62</td>
<td>9.15</td>
<td>9.84</td>
</tr>
<tr>
<td>All</td>
<td>19.95</td>
<td>30.31</td>
<td>31.58</td>
<td>32.52</td>
<td>33.62</td>
<td>35.05</td>
</tr>
<tr>
<td>Skin</td>
<td>6.70</td>
<td>10.13</td>
<td>10.75</td>
<td>11.32</td>
<td>12.10</td>
<td>13.02</td>
</tr>
</tbody>
</table>

Test data: 512x512x512  
Pure random: 1M random accesses  
Cell-wise (all): all cells  
Cell-wise (skin): cells classified as skin  
43M voxels
Volume Navigation System
Sample Slices (I)

Uncompressed
720MB
48.98MB
38.27MB
26.31MB

14.7 : 1
18.8 : 1
27.4 : 1
Sample Slices (II)

Uncompressed
720MB

14.7 : 1
48.98MB

18.8 : 1
38.27MB

27.4 : 1
26.31MB
Ray-Cast Images (I)

Uncompressed
720MB

14.7 : 1
48.98MB
Ray-Cast Images (II)

Uncompressed
720MB

18.8 : 1
38.27MB
Ray-Cast Images (III)

Uncompressed
720MB

27.4 : 1
26.31MB
Ray-Cast Images (IV)

11.3 : 1
63.69MB

14.7 : 1
48.98MB
Ray-Cast Images (V)

18.8 : 1
38.27MB

27.4 : 1
26.31MB
Conclusions

• An effective 3D compression scheme for very large volume data
  ▪ 3D wavelet-based compression (multi-resolution)
  ▪ Fairly fast random access and good compression ratio
  ▪ Visualization on computer with limited memory
Future Works

- Performance improvement
- Application to cryosection RGB images
- Test with other filters
- Application development
  - Interactive virtual navigation for human body
  - Parallel volume rendering
  - 3D fusion of CT/MRI and RGB data
  - Etc.
Parallel Volume Rendering on CRAY T3E

- Distributed memory parallel computer
- SHMEM (Shared Memory Access routines)
- Based on our 3D encoding scheme
- Ray casting
- Image space partition
- No need for volume data redistribution
Task Partition on Image Plane
Sample Run